

Noise-invoked resonances near a homoclinic bifurcation in the glow discharge plasma

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(Received 7 March 2008; published 14 August 2008)

Stochastic resonance (SR) and coherence resonance (CR) have been studied experimentally in discharge plasmas close to a homoclinic bifurcation. For the SR phenomenon, it is observed that a superimposed subthreshold periodic signal can be recovered via stochastic modulations of the discharge voltage. Furthermore, it is realized that even in the absence of a subthreshold deterministic signal, the system dynamics can be recovered and optimized using noise. This effect is defined as CR in the literature. In the present experiments, induction of SR and CR is quantified using the absolute mean difference and normalized variance techniques, respectively.

DOI: [10.1103/PhysRevE.78.026406](https://doi.org/10.1103/PhysRevE.78.026406)

PACS number(s): 52.80.Hc, 05.40.-a, 05.45.Xt, 05.20.-y

I. INTRODUCTION

Stochastic resonance (SR) is a phenomenon in which the response of a nonlinear system to a weak periodic input signal is amplified or optimized by the presence of a particular level of noise [1], i.e., a previously untraceable subthreshold signal applied to a nonlinear system can be detected in the presence of noise. Furthermore, there exists an optimal level of noise for which the most efficient detection takes place [2,3]. SR has been observed in many physical, chemical, and biological systems [2–9]. Coherence resonance (CR) is the phenomenon wherein regularity of the dynamical behavior emerges by virtue of the interplay between the autonomous nonlinear dynamics and the superimposed stochastic fluctuations. In CR, analogous to SR, the extent of provoked regularity depends upon the amplitude of added noise [10,11]. The CR effect too has been studied exhaustively, both theoretically and experimentally, in a wide range of nonlinear systems [12–17].

The nonlinearity in plasma systems arises from the most fundamental processes, namely, the wave-wave and wave-particle interactions. Different modes may be excited due to nonlinear coupling of waves and plasma components and the character of the oscillations is primarily determined by the plasma parameters and perturbations [18–20]. In the present work, the possibility of observing noise-invoked resonances in a glow discharge plasma is explored. It is hoped that the observation of CR and SR in the glow discharge plasma will lead to a better understanding of the plasma dynamics. As a precursor to the experiments involving noise, a systematic analysis of the autonomous dynamics is performed. This includes identification and characterization of the bifurcation in the vicinity of the set point employed for the noise-related experiments.

II. EXPERIMENTAL SETUP

The experiments were performed in a hollow cathode dc glow discharge plasma. The schematic diagram of the experimental setup is presented in Fig. 1. A hollow stainless steel tube of length ≈ 7 cm and of diameter (ϕ) ≈ 45 mm was used as the cathode and a central rod of length ≈ 7 cm and $\phi \approx 1.6$ mm was employed as the anode. The whole assembly was mounted inside a vacuum chamber and was pumped down to a pressure of about 0.001 mbar using a rotary pump. The chamber was subsequently filled with argon gas up to a predetermined value of neutral pressure by a needle valve. Finally a discharge was struck by a dc discharge voltage (DV), which could be varied in the range of 0–1000 V.

The noise and subthreshold periodic square pulse generators were coupled with the DV through a capacitor (Fig. 1). In all the experiments the DV was used as the bifurcation

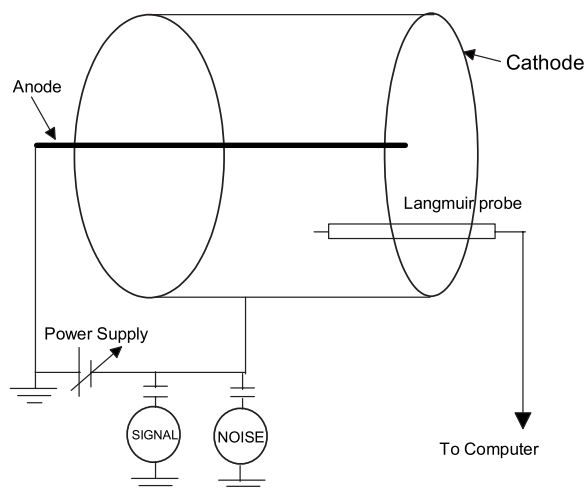


FIG. 1. Schematic diagram of the cylindrical electrode system of the glow discharge plasma. The probe was placed at a distance $l \approx 12.5$ mm from the anode. Signal and noise sources were coupled to the discharge voltage through a capacitor.

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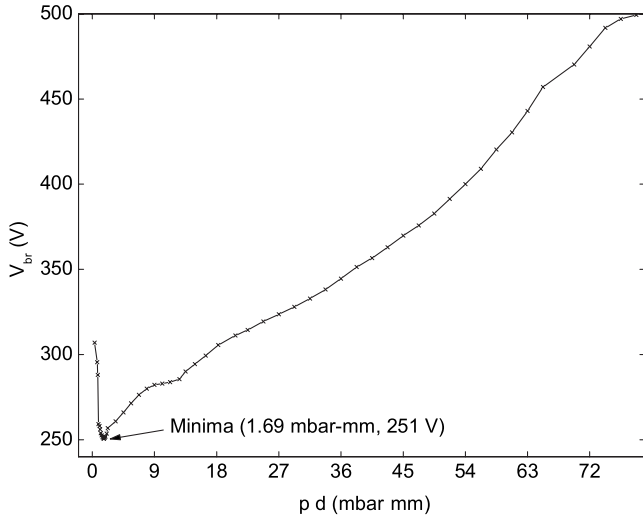


FIG. 2. V_{br} vs pd (Paschen curve) for our experimental system. The minimum occurs at (1.69 mbar mm, 251 V). The system is excitable for pd greater than the minimum of the curve.

parameter while the remaining system parameters like pressure, etc., were maintained constant. The system observable was the electrostatic floating potential, which was measured using a Langmuir probe of diameter $\phi=0.5$ mm and length 2 mm. The tip of this Langmuir probe was placed in the center of the electrode system as indicated in Fig. 1. The plasma density and the electron temperature were determined to be of the order of 10^7 cm⁻³ and 3–4 eV, respectively. Furthermore, the electron plasma frequency (f_{pe}) was observed to be around 28 MHz, whereas the ion plasma frequency (f_{pi}) was estimated to be around 105 kHz.

III. AUTONOMOUS DYNAMICS

Before studying the noise-induced dynamics, we characterized the behavior of the autonomous system. Not surprisingly, it was observed that at different chamber pressures the discharge struck at different voltages. Figure 2 shows the breakdown voltage (V_{br}) at different pd , where p and d are the filling pressure and radius of the cathode, respectively. This breakdown voltage (V_{br}) initially decreases with an increase in pd , goes through a minimum value resembling a typical Paschen curve, and then begins to increase with increasing pd . The minimum value of pd is called the Paschen minimum (Fig. 2). In the region where pd is greater than the Paschen minimum the plasma dynamics, which is measured by fluctuations in the floating potential, is irregular (complex) at the initial stages of the discharge voltage and upon increasing the DV they become regular (period 1). Further augmentation of the DV modifies the oscillation profile and results in typical relaxation oscillations [21]. These self-sustained relaxation oscillations change to excitable fixed point behavior with increase in the DV. The DV at which these oscillations cease may be termed the bifurcation point (V_H), and V_H depends upon the neutral pressure. The higher the neutral pressure, the higher the V_H .

The oscillations in floating potential are related to the appearance of the anode glow near the anode [21]. The poten-

tial of the anode glow is positive in nature and the height is of the order of the ionization potential [21–23]. The relaxation oscillations can be attributed to the formation of a double layer and has been discussed in detail in Ref. [21]. An estimate of the frequency of these oscillations can be obtained from the ion transit time in the plasma [24,25] $\tau(d)=d/V_{th,i}=d/\sqrt{k_b T_i/m}$, where d is the electrode distance. The estimated ion transit frequency ($1/\tau$) for our experimental system is of the order of a few kilohertz, which agrees well with the frequency of the relaxation oscillations.

The time period (T) of these relaxation oscillations increases dramatically upon further incrementing the DV. This eventually results in the vanishing of the limit cycle behavior beyond the critical DV (V_H). For larger values of the DV, the autonomous dynamics exhibits a steady state fixed point behavior. Time traces from top to bottom in the left panel of Fig. 3 depict this period lengthening of the oscillatory behavior. A systematic analysis of the increment in the period (T), presented in Fig. 3(a), right panel, indicates that the autonomous dynamics undergoes a critical (exponential) slowing down. Consequently, the $\ln|V-V_H|$ vs T curve can be fitted by a straight line, where V_H is the bifurcation point separating the oscillatory domain and the steady state behavior. The results of Fig. 3 indicate that the system dynamics undergoes a homoclinic bifurcation at V_H resulting in the loss of oscillations.

IV. NOISE-INVOKED DYNAMICS

In this section experimental results involving noise-generated resonances, namely, SR and CR, are presented.

A. Stochastic resonance

For our experiments on stochastic resonance, the reference voltage V_0 was chosen such that $V_0 > V_H$ and therefore the autonomous dynamics, by virtue of an underlying homoclinic bifurcation, exhibits steady state behavior. The discharge voltage V was thereafter perturbed, $V=V_0+S(t)+D\xi$, where $S(t)$ is the subthreshold periodic pulse train chosen for which $V=V_0+S(t) > V_H$ (the subthreshold signal does not cause the system to cross over to the oscillatory regime), and $D\xi$ is the added Gaussian white noise ξ with amplitude D . A subthreshold periodic square pulse of width 20 μ s and duration 2 ms was constructed using a Fluke PM5138A function generator. Meanwhile, the Gaussian noise produced using the HP 33120A noise generator was subsequently amplified using a noise amplifier.

Figures 4(a)–4(c) show time series of the system response in the presence of an identical subthreshold signal for three different amplitudes of imposed noise. The subthreshold periodic pulse train is also plotted, in the topmost graph of the left panel, for comparison purposes. Figure 4(a) shows that there is little correspondence between the subthreshold signal and the system response for a low noise amplitude. However, there is improved correspondence at an intermediate noise amplitude [Fig. 4(b)]. Finally, at higher amplitudes of noise the subthreshold signal is lost amid stochastic fluctuations of

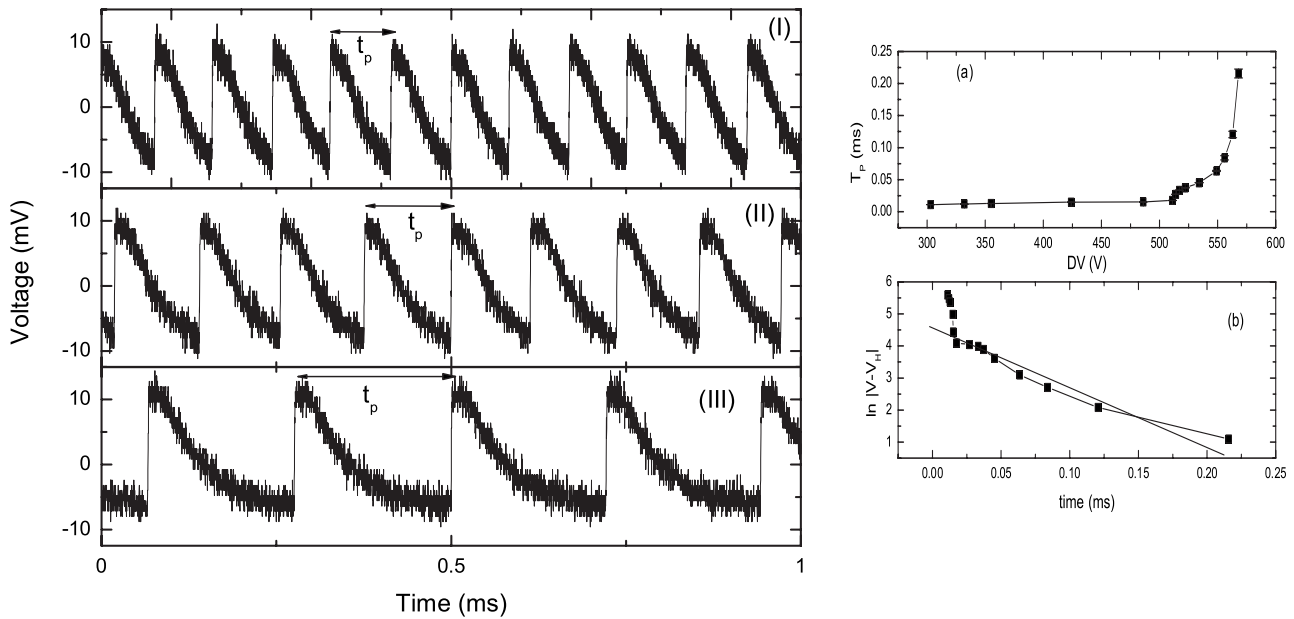


FIG. 3. Left panel: Time series showing that the period of relaxation oscillations increases with augmenting DV. Right panel: (a) Exponential increment of the time period (T) with DV and (b) $\ln|V-V_H|$ vs T curve fitted by a straight line indicating an underlying homoclinic bifurcation.

the system response [Fig. 4(c)]. The absolute mean difference (AMD), used to quantify the information transfer between the subthreshold signal and the system response, is defined as $AMD = \text{abs}[\text{mean}(t_p/\delta - 1)]$. t_p and δ are the interpeak interval of the response signal and mean peak interval of the subthreshold periodic signal, respectively. Figure 4(d) shows that the experimentally computed AMD versus noise amplitude D curve has a unimodal structure typical for the SR phenomenon. The minimum in this curve corresponds to

the optimal noise level for which maximum information transfer between the input and the output takes place.

B. Coherence resonance

For the experiments on coherence resonance, the DV (V_0) was located such that the floating potential fluctuations exhibit fixed point behavior. In order to minimize the effect of parameter drift, a set point (V_0) quite far from the homoclinic

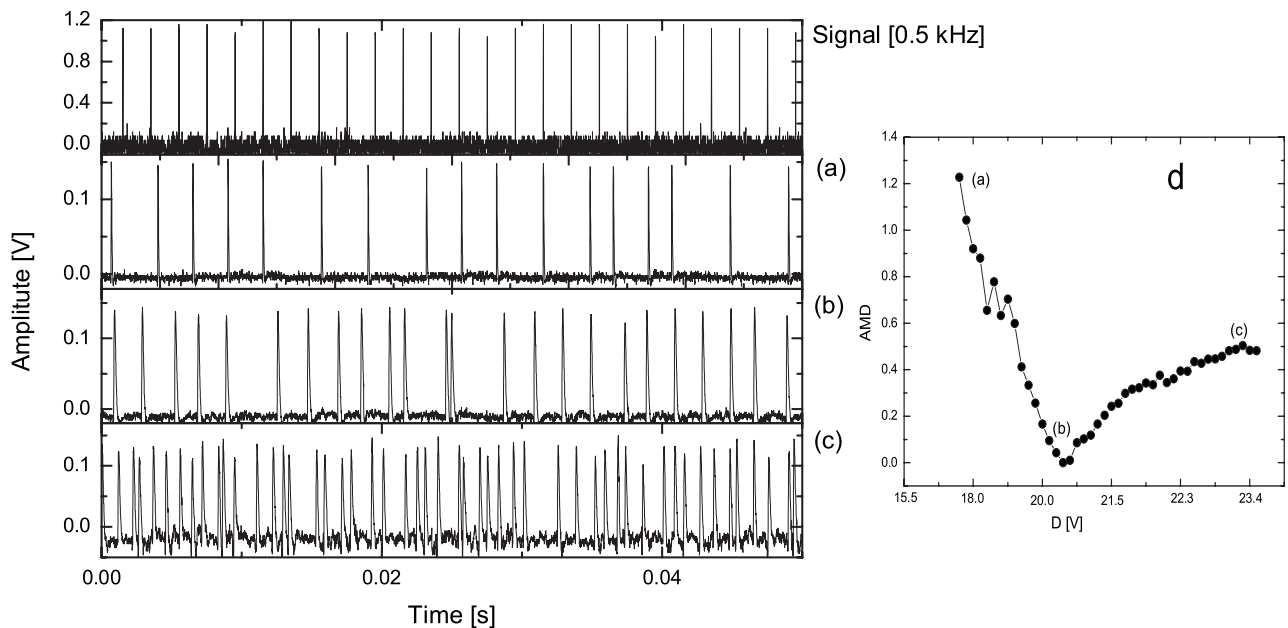


FIG. 4. Noise-induced system response in the floating potential fluctuations for low-, medium-, and high-amplitude noise in conjunction with a subthreshold periodic square pulse. Right panel: shows the AMD as a function of noise amplitude for the experiment performed at $V_0=307$ V and pressure 0.39 mbar. Left panel: subthreshold periodic pulse train and the three time series of floating potential fluctuations at low-level (a), optimum (b), and high-amplitude (c) noise.

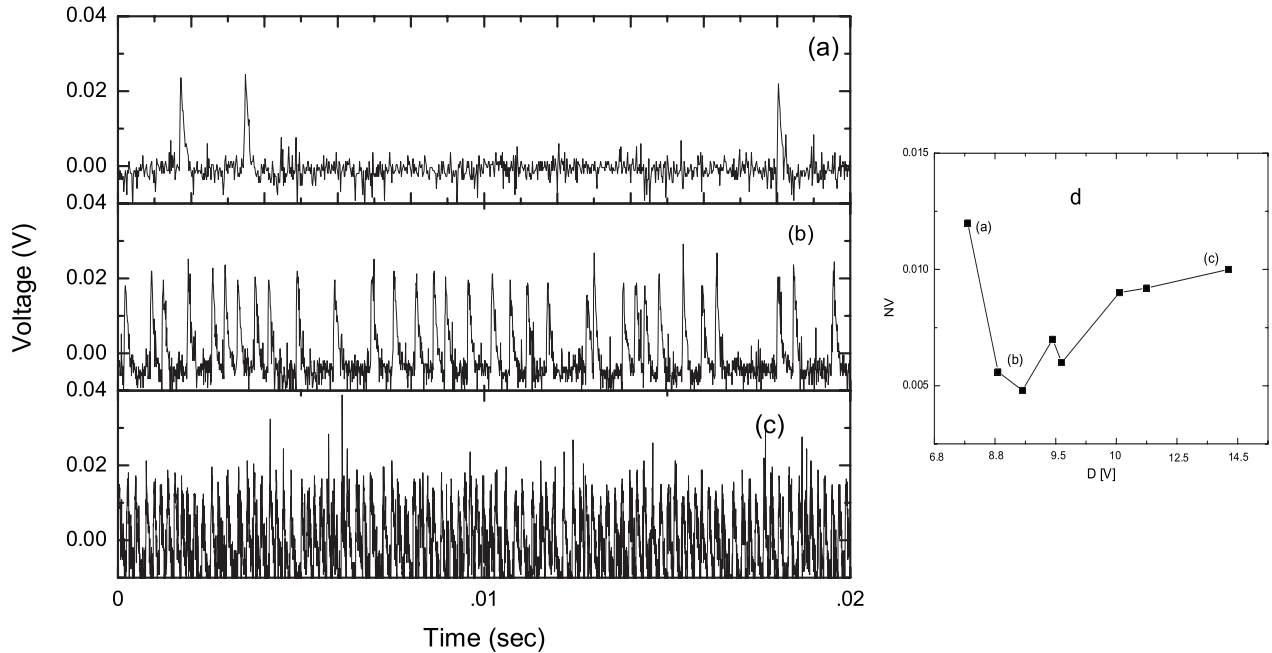


FIG. 5. Emergence of coherence resonance: The right panel shows the NV as a function of noise amplitude for the experiments performed at $V_0=344$ V and pressure 0.5 mbar. Left panel: The time series of the floating potential fluctuations for (a) low, (b) optimum, and (c) high-level noise

bifurcation (V_H) was chosen. Subsequently, the noise superimposed on the discharge voltage was increased and the provoked dynamics analyzed. The normalized variance (NV) was used to quantify the extent of induced regularity. It is defined as $NV = \text{std}(t_p) / \text{mean}(t_p)$, where t_p is the time elapsed between successive peaks. It is evident that the more regular the induced dynamics the lower the value of the computed NV. For purely periodic dynamics the NV goes to zero.

Figures 5(a)–5(c) (left panel) show the time series of the floating potential fluctuations for different noise levels and Fig. 5(d) (right panel) is the experimental NV curve as a function of noise amplitude D . The point (a) in Fig. 5(d) [time series shown in Fig. 5(a)] is associated with a low level of noise where the activation threshold is seldom crossed, generating a sparsely populated irregular spike sequence. As the noise amplitude is increased, the NV decreases, reaching a minimum (b) in Fig. 5(d) [time series shown in Fig. 5(b)] corresponding to an optimum noise level where maximum regularity of the generated spike sequence is observed. As the amplitude of superimposed noise is increased further, the observed regularity is destroyed manifested by an increase in the NV; label (c) in Fig. 5(d) [time series shown in Fig. 5(c)]. This is a consequence of the dynamics being dominated by noise.

V. DISCUSSION

The effect of noise has been studied experimentally near a homoclinic bifurcation in a glow discharge plasma system. Our study demonstrates the emergence of SR for periodic subthreshold square pulse signals and the induction of CR via purely stochastic fluctuations. In SR experiments, the efficiency of information transfer was quantified using AMD

instead of the power norm which has been utilized elsewhere [3]. The advantage of using this method in comparison to the power norm $[C_0(0)]$ [3] lies in the fact that AMD remains independent of the lag between the measured floating potential and the applied periodic square pulse. This is of relevance to our experimental system, where invariably there exists a lag, at times varying in time due to the parameter drifts. For the CR experiments it was occasionally observed that, while with an initial increase in noise amplitude (D) NV reaches a minimum, the subsequent rise of NV for even higher amplitudes of noise was suppressed. This leads to the modification of the unimodal profile, a signature of the CR phenomenon. A possible explanation for this suppression is that by virtue of the superimposed high-frequency noise (bandwidth 500 kHz) and fast-responding internal plasma dynamics, the system has the capability of exciting high-frequency regular modes within the ion plasma frequency (105 kHz). This in turn leads to the persistence of low NV values. Finally, in Refs. [12,13] both the destructive and constructive roles of noise (CR only) have been reported for glow discharge and magnetized rf discharge plasma systems, respectively. However, both these experiments were carried out in the vicinity of the Hopf bifurcation. In contrast, for the present work we studied both stochastic and coherence resonance in the neighborhood of the homoclinic bifurcation.

ACKNOWLEDGMENTS

This work has been done under Indo-Mexican bilateral project supported by DST (India) and CONACyT (Mexico). The authors acknowledge A. Bal and the Micro Electronic Division of SINP for their technical help during the experiments.

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